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Regression Diagnostics in Gamma Regression and Partial Residual Plots

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Abstract
This study describes partial residual plots with its structure and utility in the generalized linear model (GLM) setting. These plots are used as a tool for visualising diagnostics and curvature as a function of chosen predictors. In this case, partial residual plots are constructed and gamma regression as a GLM is taken into consideration. Depending on how the response variable behaves and how the affiliated link function interacts with various covariates, these graphs may or may not be effective at providing a clear visual representation of curvature and diagnostics. We investigate the behaviour of the population version, the estimated coefficients, and the partial residuals. For improved perception of fit issues, specification issues, and data abnormalities, many diagnostics in a single display are available.

Keywords: Visual Impression; Gamma regression; diagnostics; Predictor transformations.

Introduction
Statistical graphics play an important role in every stage of analysis. The best graphics are used to convey complex information simply, effectively, and clearly. They are frequently employed for model refinement and diagnostics, for the storage and recovery of data summaries, and for the clear, effective presentation of results. They also help connect data set combinations for different variables,[Jacoby, 1997; Natrella, 2011].
Initially, Ezekiel [1924] and then Larsen and McCleary[1972] used partial residual plots and their generalization, named as augmented partial residual plot is proposed by Mallows [1986] and later Mansfield and Conerly[1987] made illustration of using partial residual and conventional residualjointly. Cook [1993] extended this study and obtained CERES plots by using conditional expectation.
For the purpose of visualising unknown function and recognising the necessity to change predictors in regression with binary response, Landwehr et al. [1984] constructed and employed partial residual plots in GLM context. For generalized linear models (GLM), McCullagh and Nelder [1983] introduced the generalized partial residual plot.Fowlkes[1987] subsequently modified these plots by applying a smoother for logistic regression to evaluate nonlinearity in explanatory variables.Santner and Duffy [1989] and Collett [1991] introduced partial residual plots in logistic framework.Using added-variable

plots, Wang [1985] used a different strategy for inclusion of a specific explanatory variable in a model. Cook and Cross-Dabrera [1998] provided construction and significance of partial residual plots to diagnose curvature and studied stochastic behavior of explanatory variables in a model. According to Chatterjee et al. [1986], partial residual plots may be used to diagnosis regression model, specification error, outlier, multicollinearity, nonlinearity, hetrocedasticity, and other issues that arise in the model in addition to helping students diagnose the value in graphical form.According to Stine Robert [1995], partial residual plots and regression plots are two graphical tools used to identify problems with multicollinearity, hetrocedasticity, and outliers as well as to see how important observations affect the data. For simple linear models and logistic regression models, Pregibon [1981] is employed.When an appropriate transformation is required for the nonlinear regression model, Davision and Tsai [1992] provided the very common definition of a partial residual plot and described how it is employed.The partial residual plots for weighted regression models were created by Hines and Carter [1993].

Partial residual plots have a wide range of applications, including medicine [Saulnier et al., 2018], as well as evaluating various metrics for measuring the performance of the plant community for various ecosystem services on green roofs [Xie et al., 2018]. Other applications can be seen in Wouters et al. [2018], etc.

By employing the classical Gamma regression model, McCullagh and Nelder [1989] established the Gamma regression model and determined the mean and dispersion of the regression coefficient.Lunde and Asger [1999] discussed the use of Generalized Gamma autoregressive model in the price duration data, estimate the time required in the price changing data.

Generalized linear models were used in censored data by Pascoa et al. [2003] to model real-world data in the context of some core distributions. The weibull and gamma distributions, as well as other important models that use the reciprocal of the distribution, were also used to explain the error distribution for log Gamma generalised linear models. Skarohamar et al. [2010] faced numerous types of heterogeneity in the data when using the generalised linear mixed model to examine the factors contributing to the rising crime rate. This model is also employed when measuring the random effect in distributions, which is typically done when determining whether a distribution belongs to a continuous or discrete distribution. In a set of gamma regression models that Bossio et al. [2015] described, the response variable has a gamma distribution, and the class of predictors that make up its mean are a linear regressor function. In this case the form parameter of a gamma regression model depends on the type of regressor that is produced by a link function or a logarithm function. They claimed that the gamma regression model has several applications in heterogeneous insurance in the rate-setting process and that it performs a crucial role in insurance. Positive random variables are studied using the gamma distribution.

Johnson [2014] described the importance of Gamma distribution in generalized linear model and explained it cannot be proved with example but it can be explained by use of Gaussian distribution meaningful then the Gamma distribution in GLM. He stated that the Poisson and negative binomial distributions provide distinct results, but the Gamma distribution does not. Edilberto et al.[2016] described the Gamma regression model in detailed and obtain the residuals in which shape parameter and mean follow the Gamma regression model also they are used the Bayesian and classic technique in the Gamma regression model, to check the important of the residual in which they are used the simulated and real life data.

In Literature, various researchers used partial residual plots in different situations under multiple regression analysis. Larsen and McCleary [1972] used these plots to detect the linearity, hetroscedasticity, outlier and described the curvilinear linear relationship. Cook [1993] also used these plots for the regression diagnostics in the curvilinear problem. For checking linearity, heterogeneity and improvement of model fitting in multiple linear regressions, Huynh [2000] also used the same techniques. Furthermore, the partial residual plots only constructed in GLMs for logistic regression.

The current study uses various residuals from the Gamma regression model to illustrate the construction and diagnostics of partial residuals plots.The structure of the essay is as follows. We demonstrate the creation of partial residual plots for the gamma regression in section 2. We provided a numerical illustration of the Gamma regression model of Scottish electoral politics on September 11, 1997 [Gill, 2000] in section 3. The analysis and use of partial residual plots for gamma regression are also covered in this section. In part 4, we summarized the conclusions.

In Gamma Regression, Exploring Partial Residual Plots

The construction of partial residual plots can be seen in, Landwehr et al. [1984], Imran and Akbar [2020], and Hussain and Akbar [2022].As follows are descriptions of the GLM:

$$g(\mu_i) = \acute{x}_i\, \beta = \eta \qquad (1)$$

Let the response variable "Y" belongs to the family of exponential distributions; the conditional distribution of "Y" given "X" is [Imran, 2022].

$$d_{y|x}(y|\theta,\psi) = exp\left\{\frac{(y\theta-\mu(\theta))}{\upsilon(\psi)} + \omega(y,\psi)\right\}, \qquad (2)$$

While such a (.), b (.), and c (.) be the smooth functions that depends on X.

$E(Y/X) = \frac{\partial\mu}{\partial\theta} = \mu(x)$andvariance function is$\left\{\frac{\partial^2\mu}{\partial\theta^2}\right\}\nu(\psi)$.

Cook and Cross-Debrara [1998] established a precedent.We partition$\boldsymbol{X'} = (\boldsymbol{X_1'}, \boldsymbol{X_2'})$where X_j be,$p_j \times 1$, $j = 1, 2$. With presumption that the structure correctly characterises the regression equation.

$$\eta(x) = h\big(\mu(x)\big) = \alpha_0 + \boldsymbol{\alpha'}_1 x_1 + g(x_2),$$

If such term is evaluated parametrically, therefore;

$$\eta(x) = h\big(\mu(x)\big) = \alpha_0 + \boldsymbol{\alpha'}_1 x_1 + \boldsymbol{\alpha'}_2 x_2\,, \qquad (3)$$

Therefore$h$(.) be monotonic and differentiable specific link function, $(\alpha'_0, \boldsymbol{\alpha'}_1, \boldsymbol{\alpha'}_2)'$be vector containing parameters to be estimated having order$(p_1 + 1) \times 1$. Whereexpression$\mu(x) = h^{-1}\big(\eta(x)\big)$,be a function that depends on x or η.

The probability density function (PDF) of Gamma distribution is given as;

$$f(y;\alpha,\beta) = \frac{1}{\beta^{\alpha}\,\alpha} y^{\alpha-1} e^{-\frac{y}{\beta}}, \qquad y \geq 0,\ \alpha,\beta > 0 \qquad (4)$$

Given below are the values for y's mean and variance, respectively.

$$E(y) = \alpha\beta \,\&\, var(y) = \alpha\beta^2.$$

Eq. (4) can be transformed using parameters $\alpha = \frac{1}{\phi}$&$\beta = \mu\phi$, proposed Hardin and Hilbe (2012).

Gamma density for y is

$$f(y;\mu,\phi) = \frac{1}{\Gamma\left(\frac{1}{\phi}\right)}\left(\frac{1}{\mu\,\phi}\right)^{\frac{1}{\phi}} y^{\frac{1}{\phi}-1}\, e^{-\frac{y}{\mu\,\phi}}, \qquad y \geq 0,\ \mu > 0,\ \phi > 0. \quad (5)$$

Also worth mentioning is that y's mean and variance are defined as follow,

$E(y) = \mu \,\& var(y) = \phi,\ var(\mu) = \phi\mu^2.$

via (2), the (5) are as explained;

$$f_{y/x}(\mu_i\,,\phi) = exp\left\{\frac{-\frac{y_i}{\mu_i}-(-(ln\mu_i))}{\phi} + \frac{1-\phi}{\phi}\, ln(y_i) - \frac{ln(\phi)}{\phi} - ln\left(\Gamma\left(\frac{1}{\phi}\right)\right)\right\} \qquad (6)$$

Where, $\theta = -\frac{1}{\mu}\,,\ b(\theta) = ln(\mu)\,,\ a(\phi) = \phi\,,$

$$and c(y,\phi) = \frac{1-\phi}{\phi} ln(y_i) - \frac{\ln(\phi)}{\phi} - ln\left[\Gamma\left(\frac{1}{\phi}\right)\right]$$

by (1) and (5) the link function isdefinedby;

$$\eta = \theta = h\,(\mu) = \frac{-1}{\mu}. \qquad (7)$$

Therefore,

$$\mu(\eta) = \frac{-1}{\mu}$$

Towards proceeding the estimation of unknown parameters, we are using l_i for log of likelihood of the response variable. Then (6) becomes;

$$l_i = l_i(\mu_i , \phi) = \sum_{i=1}^{n} \left\{ \frac{-\frac{y_i}{\mu_i} - \left(-(ln(\mu_i))\right)}{\phi} + \frac{1-\phi}{\phi} ln(y_i) - \frac{ln(\phi)}{\phi} - ln\left(\Gamma\left(\frac{1}{\phi}\right)\right) \right\} \quad (8)$$

Suppose$\hat{\beta},\ \hat{\mu},\ and\ \hat{\phi}$ areMLEswhich can be achieved by Netwon-Raphson iterative method.Iterative Netwon-Raphson methods are employed to estimate the unknown parameter. Hardin and Hilbe (2012)provided the starting values and complete technique for the estimation of the unknown parameter for the iterative Gamma regression model.

Then, by; follows the fitted Gamma regression model.

$$\eta_f(x|\hat{\mathbf{b}}) = h(\hat{\mu}_f) = (\hat{b}_0 + \hat{\mathbf{b}}'_1 x_1 + \hat{\mathbf{b}}'_2 x_2)^{-1} \quad (9)$$

Therefore,$\hat{\mathbf{b}}' = (\hat{b}_0, \hat{\mathbf{b}}'_1, \hat{\mathbf{b}}'_2)$along withthe subscript 'f' on $\eta_f$and $\mu_f$refer to the fitted model.

Coefficient estimate b_j j=0,1,2, are derivedBy minimising the convex objective function,

$$\hat{\mathbf{b}}' = (\hat{b}_0, \widehat{\mathbf{b}}', \widehat{\mathbf{b}}'_2) = argminL_N(\hat{\mathbf{b}}), \quad (10)$$

While,

$$L_N(\hat{\mathbf{b}}) = \frac{1}{N}\sum_{i=1}^{N} L\left(\eta_f(x_i|\,\hat{\mathbf{b}}, y)\right),$$

$$L_N(\hat{\mathbf{b}}) = \frac{1}{N}\sum_{1=1}^{N} L\left(\hat{b}_0 + \hat{\mathbf{b}}'_1\, x_1 + \hat{\mathbf{b}}'_2\, x_2,\ y_i\right)^{-1}.$$

While L (.,.) is an objective function that the user has specified, supposing that it is convex with reverence to its argument.The usage of maximum likelihood and ordinary least squares under (3) and (9) with a canonical link where $\theta=\eta$and specific robust estimates are at the very least included in this class, making it not very restricted.For log linear regression with the connection described in (6), the objective function in terms of maximum likelihood is;

$$L\left(\eta_f(x \mid \hat{\mathbf{b}},\ y)\right) = \{exp(\eta_f) - y\,\eta_f)\}. \quad (11)$$

The convex objective function class is an extension of the objective function class corresponding to (10),

$$L(\eta_f, y) = L(y - \eta_f),$$

Cook [1993] utilised it for additive error models (7).

Using (7) and (8) through (9) to generate a partial residual Pr2 for X_2, which is represented as follow;

$$\text{Pr2} = (y - \hat{\mu}_f)h'(\hat{\mu}_f) + \hat{\mathbf{b}}'_2 x_2 \quad (12)$$

While, $h'(.)$is the first derivative of$h(.)$ w.r.t 'μ'and$\hat{\mathbf{b}}$can be obtained by (10).

The regression function of$\hat{\mu}_f$is $\hat{\mu}_f(x) = h^{-1}(\eta_f(x|\hat{\mathbf{b}}))$ is the estimatedat$\hat{\mathbf{b}}$.

The first derivativeof Gamma link function is as follows;

$$h'(\hat{\mu}_f) = \frac{1}{\mu^2}.$$

Consequently, the fitted model for gamma regression using log-link may be written as;

$$\hat{\mu}_f = \left(\hat{b}_0 + \hat{\mathbf{b}}'_1\, x_1 + \hat{\mathbf{b}}'_2\, x_2\right)^{-1}.$$

Therefore,$\hat{\mu}_f$indicate the fitted model,$\hat{b}_0, \hat{b}_1, \hat{b}_2$ are the regression coefficients respectively andx_1, x_2 are the predictors.

The partial residual for response residual is similar to the generalization for p predictors;

$$\text{R}_{\text{Pr2}} = (y - \hat{\mu}_f)\frac{1}{\mu^2} + \hat{\mathbf{b}}'_2\, x_2$$

Additionally, the Partial residual for response residual of model having p predictorsmay expressed by;

$$\text{R}_{\text{Pri}} = (y - \hat{\mu}_f)\frac{1}{\mu^2} + \hat{\mathbf{b}}'_i x_i\,, \qquad i = 1, 2, \dots, p. \quad (13)$$

For p is the predictor as well as the fitted model is as follows:

$$\hat{\mu}_f = \left(\hat{b}_0 + \hat{\mathbf{b}}'_1\, x_1 + \hat{\mathbf{b}}'_2\, x_2 + \cdots + \hat{\mathbf{b}}'_p\, x_p\right)^{-1}, \qquad (14)$$

Using Gamma regression (4.7)

$$\left(y - \hat{\mu}_f\right) h'\left(\hat{\mu}_f\right) = \frac{y - \hat{\mu}_f}{\hat{\mu}_f},$$

The partial residuals are reduced by equation (12) as specified by Landwehr*et al*. [1984], which are alreadyprovided by Cook & Cross-Dabrera [1998], Imran & Akbar [2020], among others.

Equation (12) mayrepresented as in terms of η as;

$$\left(y - \hat{\mu}_f\right) h'\left(\hat{\mu}_f\right) = \frac{\partial\, log\, d_{y|x} \mid \partial\eta}{-E\left[\partial^2\, log d_{y|x} \mid \partial\eta^2\right]},$$

Consequently $E\left[\,\partial\, log d_{y|x}|\,\partial\eta\right] = 0\ \ and\ \ -E\left[\partial^2 log d_{y|x}|\partial\eta^2\right] = E\left[\partial log d_{y|x}|\partial\eta\right]^2,$

An expression$\left(y - \hat{\mu}_f\right) h'\left(\hat{\mu}_f\right)$may describedin terms of its standard deviation as,

Consider,

$$\hat{\eta}_f(x) = \eta_f\left(x \mid \hat{\mathbf{b}}\right),$$

Therefore,

$$\hat{\eta}_f + \left(y - \hat{\mu}_f\right) h'\left(\hat{\mu}_f\right)$$

can be used to estimate the iterative estimating approach, and it is referred to occasionally as the adjusted dependent variable [Imran and Akbar 2020].The form of residual as indicated in (12) may also be investigated in the works of McCullagh and Nelder [1989], Imran and Akbar [2020], etc.

It is inspiring to realise that the phrase$\left(y - \hat{\mu}_f\right)$ in (12) and (13) corresponds to the response residual. In place of the response residual, we can substitute additional other residuals (Pearson and working) using this similarity to obtain partial residual such as Pri.

This technique is employed here by integrating Pearsonand working residuals for Gamma regression model.

Similarly,partial residual are constructed for different residual (Pearson and Working) for Gamma regression model.

Pearson Residual

According to Gill [2000], the GLM's Pearson residuals are defined as;

$$R_{Pearson} = \frac{\left(Y - \hat{\mu}_f\right)}{\sqrt{v(\hat{\mu}_i)}},$$

The observed value and fitted model are denoted in the expression above $Y$and $\hat{\mu}_f$are respectively andthe variance of$\hat{\mu}_i$is $v(\hat{\mu}_i)$.

The partial residual for the model with p explanatory variables may be written as a Pearson residual as follows:

$$\mathrm{P}_{\mathrm{Pr}i} = \frac{\left(Y - \hat{\mu}_f\right)}{\sqrt{v(\hat{\mu}_i)}}\left(\frac{1}{\mu^2}\right) + \hat{b}'_i x_i, \qquad i = 1, 2, \ldots, p. \qquad (15)$$

Working Residual

The working residual for GLM is determined by Gill [2000] as follows:

$$\mathrm{R}_{\mathrm{working}} = \left(y - \hat{\mu}_f\right)\left(\frac{\partial\eta}{\partial\mu}\right)$$

The following may be used to indicate the working residual of a model with p explanatory variables:

$$\mathrm{W}_{\mathrm{Pr}i} = \left(y - \hat{\mu}_f\right)\left(\frac{\partial\eta}{\partial\mu}\right)\left(\frac{1}{\mu^2}\right) + \hat{\mathbf{b}}'_i x_i, \qquad i = 1, 2, \ldots, p. \qquad (16)$$

Example: Electoral Politics

We used information from the 1997 vote on taxation powers for the Scottish parliament found in Gill [2000] book, as gathered from Scottish office, the general register office for Scotland, and the U.K. office for national statistics.

The response variable is percentage of Voting for the motion and the predictorssuch as, Council Tax (COU Tax), Female Unemployment (UNM), Standardized Mortality (MOR), Active Economically (ACT), and Age (AGE).

The model using (14) can be expressed as:

$$PV\ Yes = [\beta_o + \beta_1 COUTax + \beta_2 UNM + \beta_3 MOR + \beta_4 ACT + \beta_5 AGE]^{-1} + \varepsilon$$

The necessary computations as shown below.

Table 1.Gamma Regression Analysis

	Coeff.	S.E.	95% CI	VIF
Intercept	-1.7765	1.14789	[-4.14566: 0.59261]	
COU Tax	0.0049	0.00162	[0.00162: 0.00831]	2.542
UNM	0.2033	0.05321	[0.09363: 0.31326]	1.166
MOR	-0.0072	0.00271	[-0.01278: -0.00159]	1.755
ACT	0.0112	0.00406	[0.00281: 0.01956]	1.700
AGE	-0.0519	0.02403	[-0.101145: -0.00228]	1.345

The estimated regression model is given as;

$$Y = [-1.7765 + 0.0049X_1 + 0.2033X_2 - 0.0072X_3 + 0.0112X_4 - 0.0519X_5]^{-1}$$

We may generate five possible partial residuals for each variable and, consequently, six plots for each variable because the model has five variables. As a result, there might be a total of 25 plots. Fig.1 (a)-(e) shows only one plot per residual in order to simplify our presentation.

Partial residual plots are determined by:

$$\hat{P}r_i = (y - \hat{\mu}_f)h'(\hat{\mu}_f) + \hat{\boldsymbol{b}}_i' x_i$$,versus x_i

Since, $x_i = COUTax, UNM, MOR, ACT, AGE.$

And $\hat{\mu}_f = \left[\hat{b}_0 + \hat{b}_1' COUTax + \hat{b}_2' UNM + \hat{b}_3' MOR + \hat{b}_4' ACT + \hat{b}_5' AGE\right]^{-1}$

$$h'(\mu_f) = \frac{-1}{\mu^2}.$$

Consequently, utilizing the response, Pearson and Workingresiduals, we constructed plots as available in following figures.

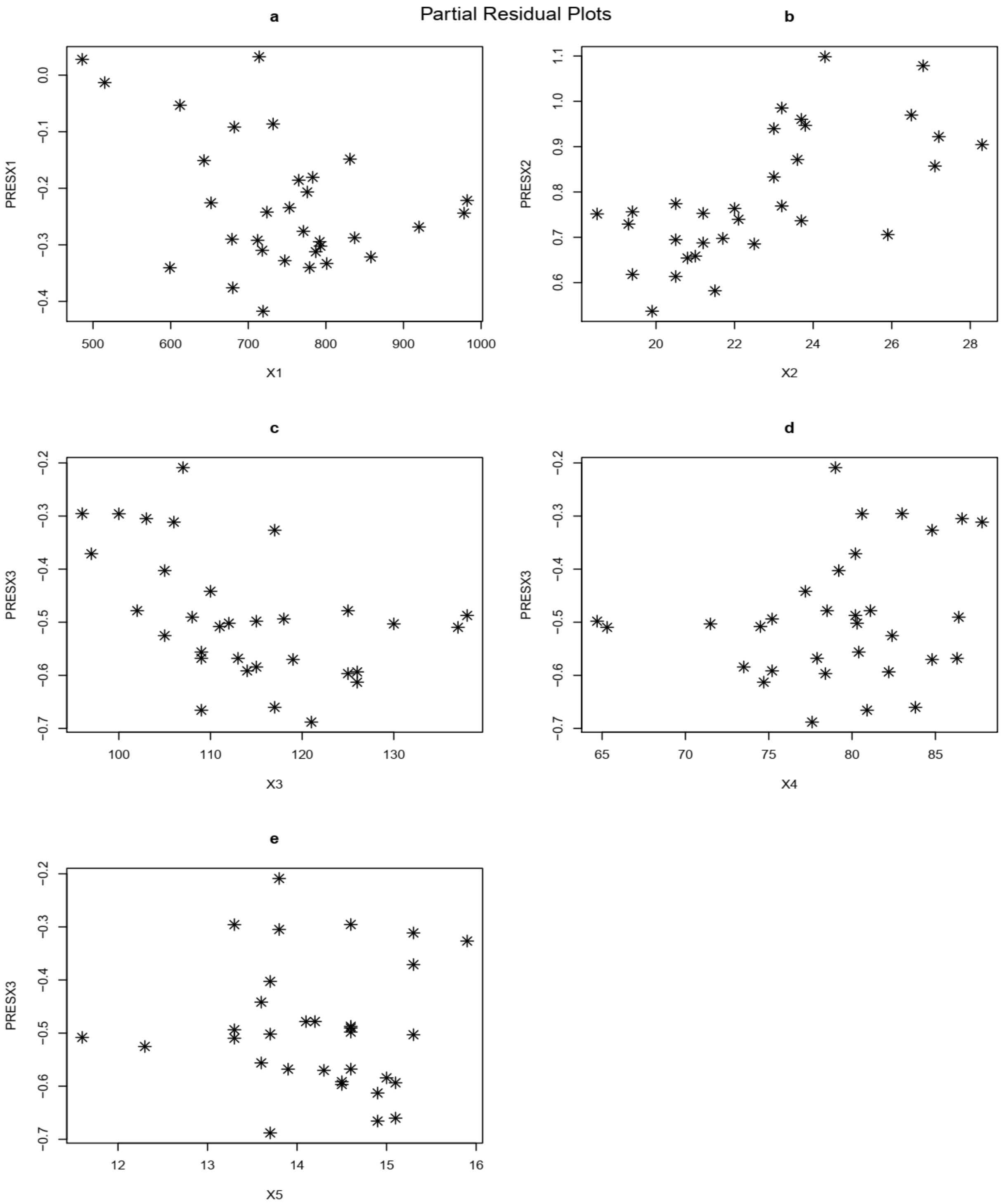

Fig. 1: Response residuals using gamma fits for partial residual plots

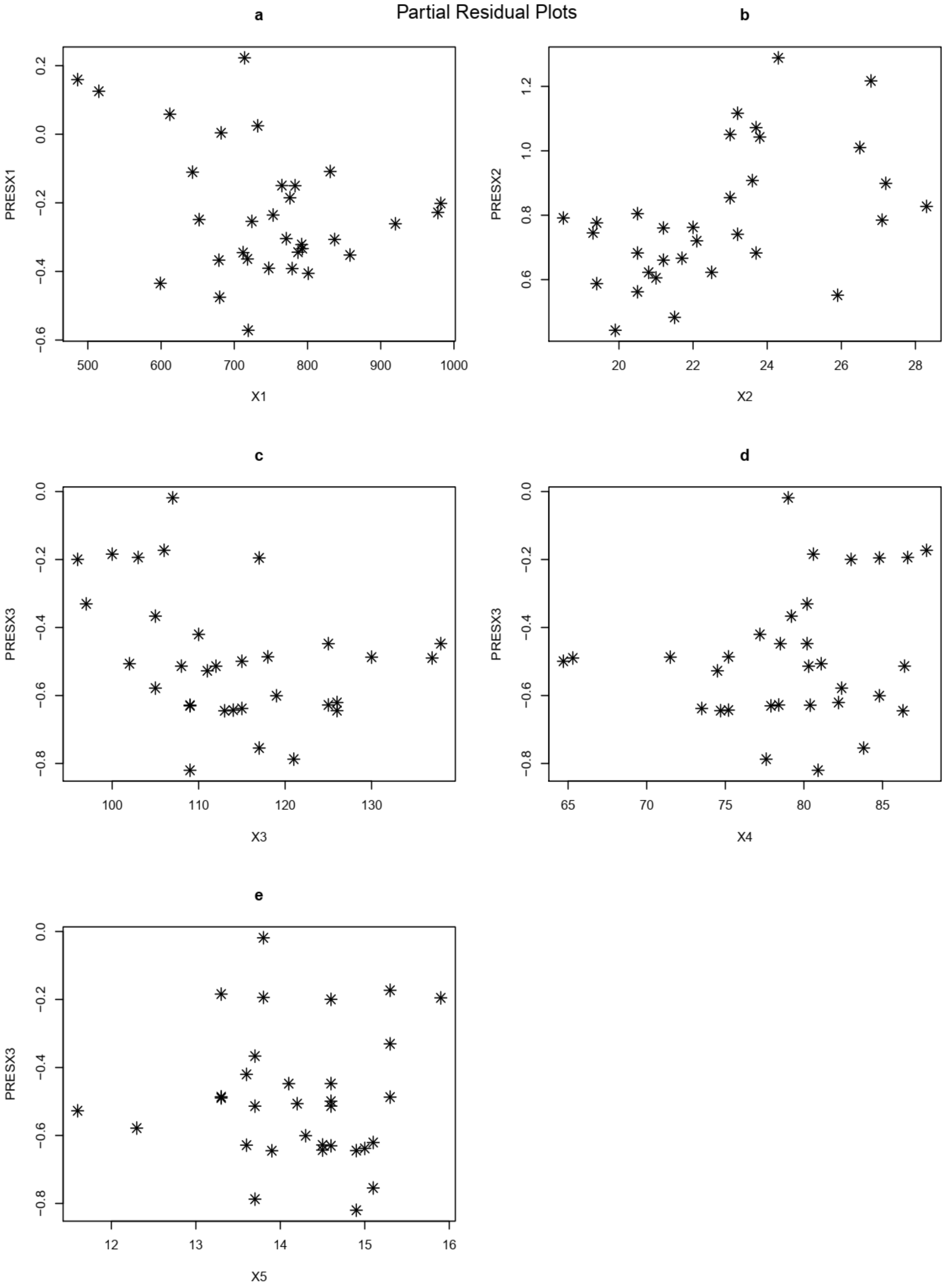

Fig. 2: Pearson residuals using gamma fits for partial residual plots.

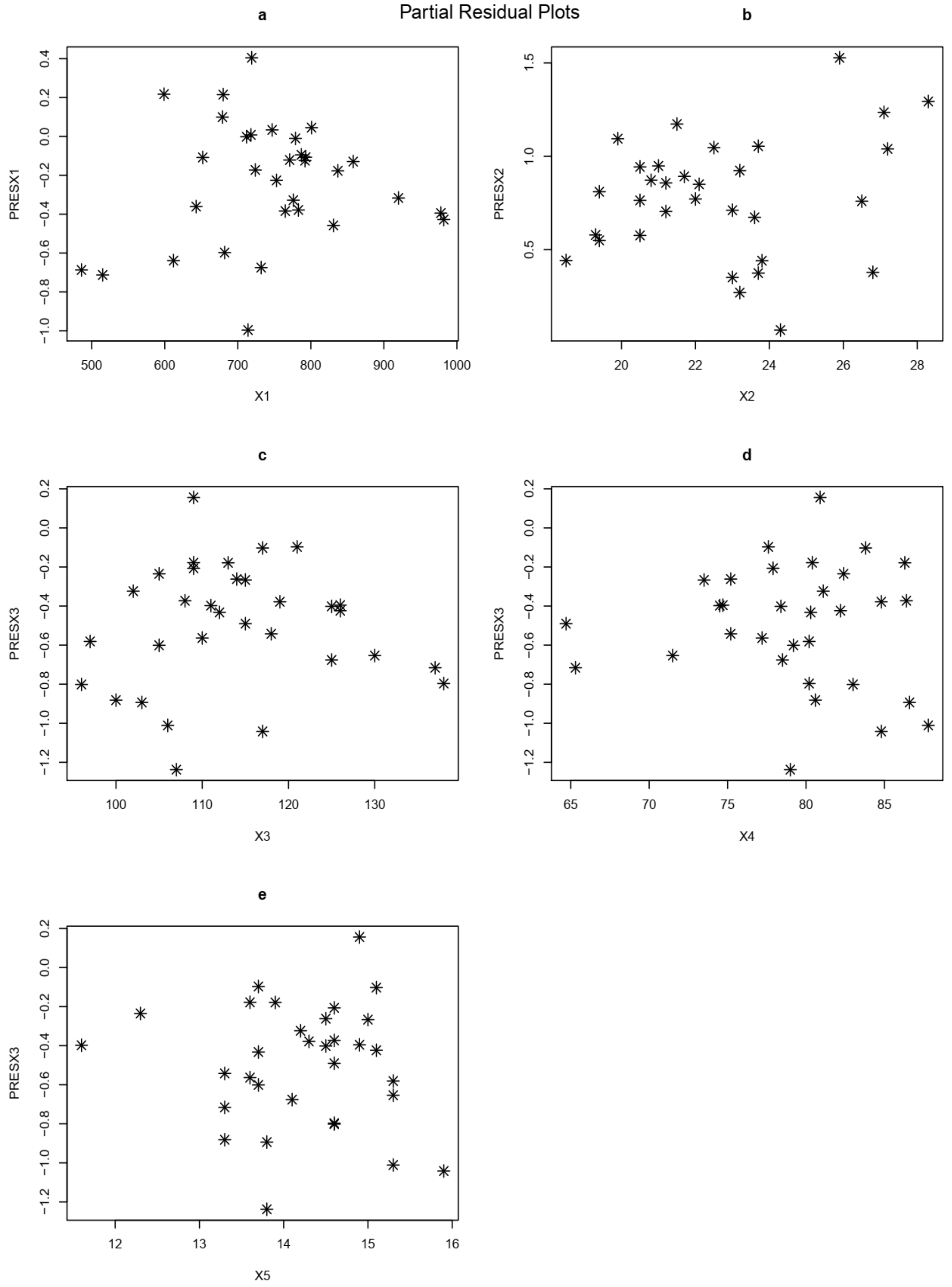

Fig.3: Working residuals using gamma fits for partial residual plots.

By employing the response residual (Fig. 1(a)), it can be seen that when the scale of the vertical axis shrinks relative to the horizontal axis, the evidence of overdispersion increases from low to high.Additionally, the partial residual plot displays wedge-shaped patterns that suggest potential nonconstant variance. These patterns can also be seen when comparing various dispersions to the value of "COU."Large embedded-image datasets are frequent, and pairwise predictor collinearity is visually notable. It is clear that one result stands out as an outlier among the rest of the data because it is far below the average. Since none of the points fall along the trend line and exhibit an irregular pattern in the residual plot, non-normality and non-linearity are expected. It may also be shown that the issues shown in Fig. 1(a) can be tracked in the remaining plots Fig. 1(b)-(e). Therefore, by employing partial residual plots in a gamma regression model, we can see several diagnostics.

The Pearson residual in Fig. 2(a)–(e) shows similar tendencies regarding the underlying data. Therefore, it can be assumed that working partial residuals reach the same diagnostics. While looking on the plots of working partial residuals, (Fig. 3(a) – (e)), the performance is same. All the Partial residual plots give the same tendencies about the data.

Now that our conclusions have been supported, we will show the diagnosis of the aforementioned issues using certain official testing techniques.

Table 2.Test for Detection of Outlier, Heteroscedasticity and Linearity

Test	Statistic	P-Value
Grubbs	5.2709	0.000
Levene	36.639	0.000
Anderson Darling	0.5641	0.133

The above table shows the presence of outliers, heteroscdasticity and linearity by respective tests. Furthermore, the multicollinearity is identified using VIF (Table 1). We have also known that there are many different formal tests available for diagnostics, each of which is based on a set of regularity criteria and necessitates additional computational work.

As the above description shows, a partial residual plot can used to diagnose multiple issues in single plot and is more efficient as compared to traditional testing.

Discussion

Partial residual plots in GLM were investigated by Cook and Cross-Dabrera [1998] and Imran and Akbar [2020], Hussain and Akbar [2022] used to evaluate the explanatory variable transformation in various regression settings.By utilising response residual in the logistic regression model and inverse Gaussian regression model, they looked at the circumstances where partial residual plots provide the helpful transformation of the predictor in GLM.The current study, working, Pearson, and response residuals are used to generate and assess partial residual plots for gamma regression. Partial residual plots are a great tool for regression diagnostics since they can quickly uncover a variety of issues.Partial residual plots are therefore advised as a useful graphical tool for the regression diagnostics in gamma regression for more than one problem at a time. Additionally, working and Pearson partial residuals perform better than the response residual and have the same diagnostic value.The specified GLM, the link function, and the predictors' stochastic behaviour might all have limitations on the usefulness of partial residual plots for generating a clear visual picture of curvature. Due to its extensive application in diagnostics employing visual impression-based prediction modification, partial residual plots were discovered to be more vivid than conventional methods.

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